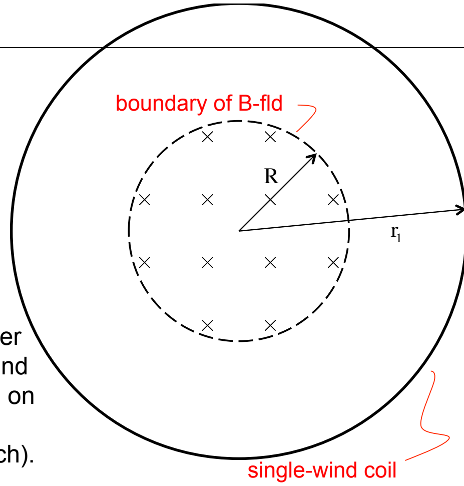
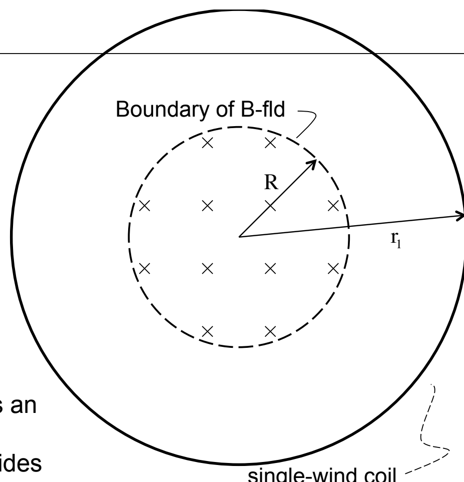
Problem 31.33

If you've looked at the title of the section from which this problem came and think you understand what's going on, don't bother to read the next two pages. Otherwise:



Before we try this problem, consider the following situation: A single-wind coil of wire of radius r_1 is centered on a circular region in which exists a changing magnetic field (see sketch).

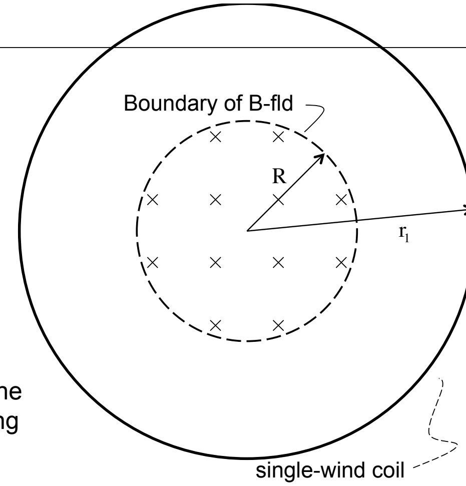
What we know from Faraday's Law: When the magnetic field changes through the face of the coil (even if the B-fld doesn't completely fill the coil's cross section), an induced EMF will be generated in the coil that will motivate an induced current to flow in the coil.



Side issue: What does an electromotive force (an EMF) REALLY do in a circuit? A battery's EMF is the part of the battery that provides energy to the circuit and that generates the ELECTRIC FIELD that motivates charge to flow.

In the situation we are examining here, there is no battery. There is an induced EMF, though, via the changing magnetic flux, that provides energy to the system and generates a circulating ELECTRIC FIELD that will motivate charge to flow.

What we are really trying to determine in this problem is the CIRCLING ELECTRIC FIELD set up by the changing magnetic flux. Determine that and we can determine the FORCE on a charge located at r_1.



An EMF from a battery shows itself as a voltage difference across the terminals of the battery. That voltage difference is related to the electric field it generates by:

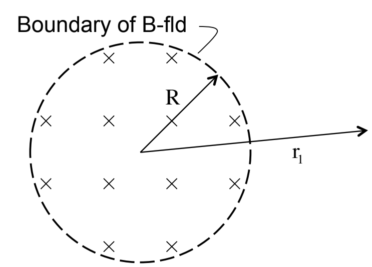
$$\Delta V = -\int \vec{E} \bullet d\vec{r}$$

In the case of the closed circuit, there is no voltage difference. The EMF is generated by the changing magnetic flux, so we can write:

$$\varepsilon = -N\frac{d\Phi_B}{dt} = \int \vec{E} \bullet d\vec{r}$$

This is the relationship we need to solve to determine the electric field.

Solving, we get:



$$N\frac{d\Phi_B}{dt} = -\int \vec{E} \bullet d\vec{r}$$

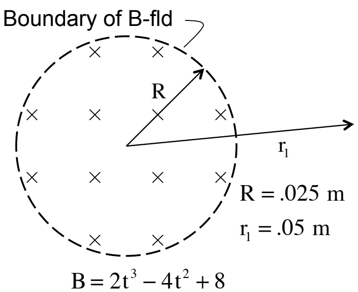
$$N\frac{d\left(BA\cos 0^\circ\right)}{dt} = -\int E\,dr\cos 0^\circ$$

$$N\frac{d\left(\left(2t^3 - 4t^2 + 8\right)\left(\pi r_1^2\right)\right)}{dt} = -E\oint dr$$

$$(1)\left(6t^2 - 8t\right)\left(\pi r_1^2\right) = -E\left(2\pi r_1\right)$$

$$\Rightarrow \quad E = \frac{\left(6t^2 - 8t\right)\left(\pi r_1^2\right)}{\left(2\pi r_1\right)}$$

a.) At t = 2 seconds:

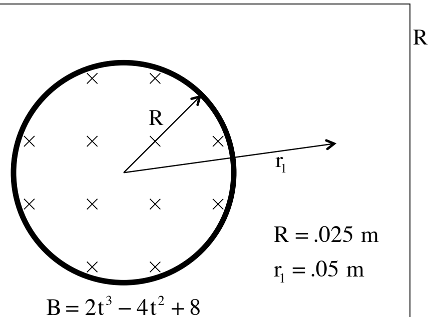


$$
\begin{aligned}
E &= \frac{\left(6t^2 - 8t\right)\left(\pi r_1^2\right)}{\left(2\pi r_1\right)} \\
&= \frac{\left(6(2)^2 - 8(2)\right)\left(\pi(.05)^2\right)}{\left(2\pi(.05)\right)} \\
&= \frac{\left(6(2)^2 - 8(2)\right)\left(\pi(.05)^2\right)}{\left(2\pi(.05)\right)} \\
&= .2 \text{ N/C}
\end{aligned}
$$

So the magnitude of the force on a charge is:

$$
\begin{aligned}
F &= qE \\
&= \left(1.6\text{x}10^{-19} \text{ C}\right)(.2 \text{ N/C}) \\
&= 3.2\text{x}10^{-20} \text{ N}
\end{aligned}
$$

b.) To get the direction of the electric field, we need to know the direction of the “induced current” generated by the changing magnetic flux. This is a job for LENZ’S LAW.

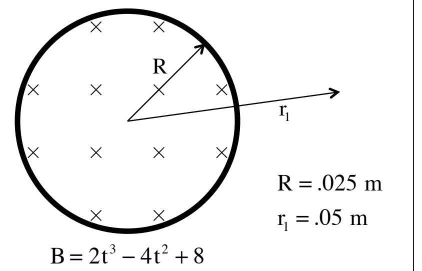


So how is the flux changing at t = 2 seconds? As the change is due to a change in the B-fld, dB/dt is what will govern this. Using that:

$$
\begin{aligned}
\frac{dB}{dt} &= \frac{d\left(2t^3 - 4t^2 + 8\right)}{dt} \\
&= \left(6t^2 - 8t\right)\Big|_{t=2} \\
&= 8 \text{ T/s}
\end{aligned}
$$

This tells us the magnetic flux is INCREASING at t = 2 seconds. An increasing magnetic flux means an induced current will generate an induced B-fld that is OPPOSITE the direction of the external field.

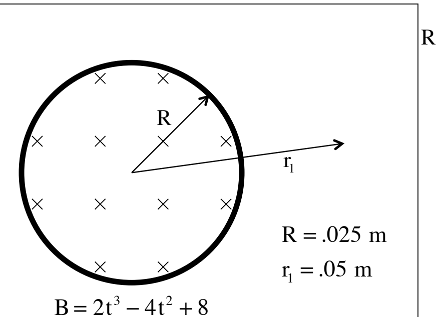
The external magnetic field is INTO the page, so the induced field is OUT OF the page.



The induced current flow that will generate the appropriate induced B-fld is COUNTERCLOCKWISE. A POSITIVE CHARGE (the kind we use for conventional current) will move in that direction (an electron will flow in the opposite direction).

Soooo, our COUNTERCLOCKWISE ELECTRIC FIELD will motivate our electron to accelerate CLOCKWISE.

c.) The force will be zero when dB/dt is zero, or when:



$$
\begin{aligned}
\frac{dB}{dt} &= \frac{d\left(2t^3 - 4t^2 + 8\right)}{dt} \\
&= \left(6t^2 - 8t\right) \\
&= 0 \text{ T/s}
\end{aligned}
$$

$$\Rightarrow \quad t = 1.33 \text{ s}$$